- MA.912.A.3.8
  Graph a line given any of the following information: a table of values, the x- and y-intercepts, two points, the slope and a point, the equation of the line in slope-intercept form, standard form, or point-slope form.

- MA.912.A.3.9
  Determine the slope, x-intercept, and y-intercept of a line given its graph, its equation, or two points on the line.

Geometry Body of Knowledge

Standard 1: Points, Lines, Angles, and Planes

- MA.912.G.1.4
  Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

Equations of Lines

An *equation* of a line can be expressed in several ways. Mathematicians sometimes use the format $ax + by = c$. This is called **standard form (of a linear equation)**. In the *standard form*, **linear equations** have the following three rules.

1. a, b, and c are **integers**, or the numbers in the set $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

2. a cannot be a **negative integer**

3. a and b cannot both be equal to 0

Linear Equations in Standard Form

- $ax + by = c$
- x and y are **variables**
- a, b, and c are **constants** for the given equation

You can graph a line fairly easily by using standard form.

Follow this example.

$$3x + 2y = 12$$

If we replace x with 0 we get the following.

$$\begin{aligned} 3x + 2y &= 12 \\ 3(0) + 2y &= 12 \\ 0 + 2y &= 12 \\ 2y &= 12 \\ y &= 6 \end{aligned}$$

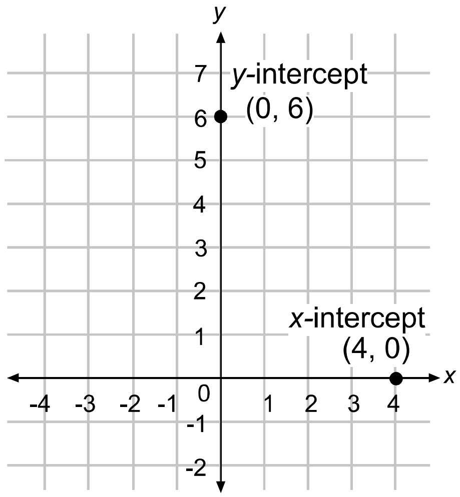
This tells us that the point (0, 6) is on the graph of the line $3x + 2y = 12$. In fact (0, 6) is called the ***y*-intercept** of the line. It is the point where the line crosses the y-axis.

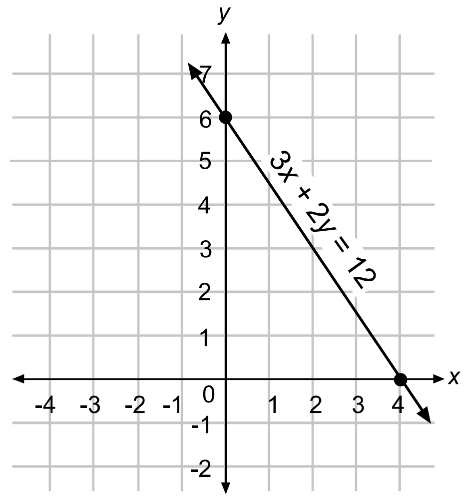
Remember that you must have two points to decide exactly where the line goes on the coordinate plane. So, we repeat the process, but this time replace y with 0.

$$\begin{aligned} 3x + 2y &= 12 \\ 3x + 2(0) &= 12 \\ 3x + 0 &= 12 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

This tells us that the point (4, 0) is also on the line. Did you guess that this is called the ***x*-intercept**?

So, if we plot the two points (0, 6) and (4, 0), we can draw a line connecting them.





Did you notice that we could find the slope of the line above either by using the slope formula with the x- and y-intercepts ($m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 6}{4 - 0} = \frac{-6}{4} = \frac{-3}{2}$) or by counting rise and run from the graph?

Let's try another example.

$$5x - y = 15$$

If $x = 0$,

$$\begin{aligned} 5x - y &= 15 \\ 5(0) - y &= 15 \\ 0 - y &= 15 \\ -y &= 15 \\ y &= -15 \end{aligned}$$

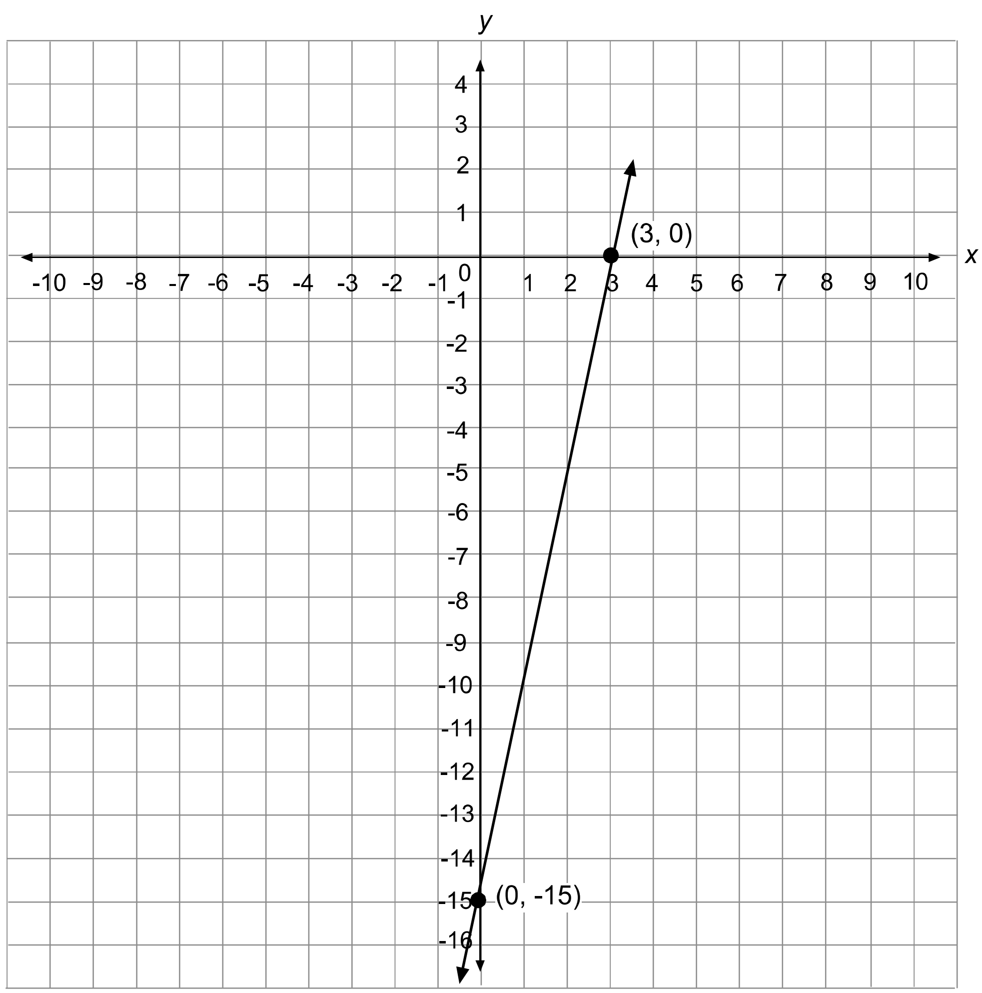
(0, -15) y-intercept

If $y = 0$,

$$\begin{aligned} 5x - y &= 15 \\ 5x - y(0) &= 15 \\ 5x - 0 &= 15 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

(3, 0) x-intercept

Graph of 5*x* – *y* = 15



Your turn.

Slope-Intercept Form

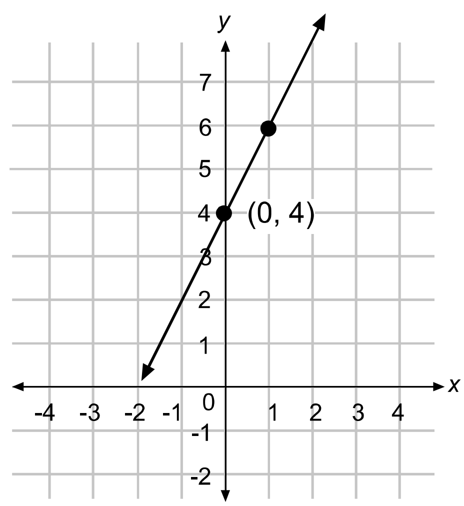
Many students prefer to use the **slope-intercept form** for the equation of a line. An equation in this form tells you the slope of a line and where it crosses the y-axis. The generic format looks like the following.

$$y = mx + b$$

m is the slope $\quad$ b is the y-intercept

So if $y = 2x + 4$, this line crosses the y-axis at 4 and has a slope of 2.

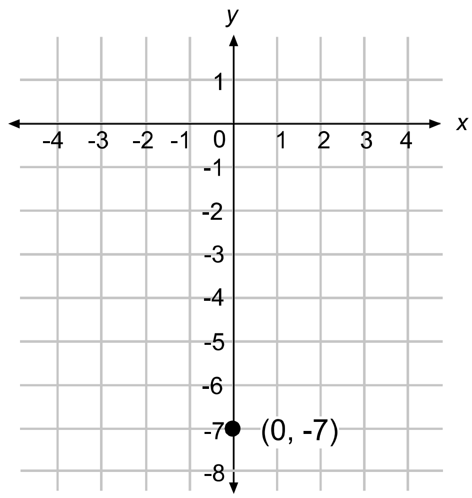
To graph this line, plot a point at (0, 4) and count the rise and run of the slope ($\frac{2}{1}$) from that point and draw a line.



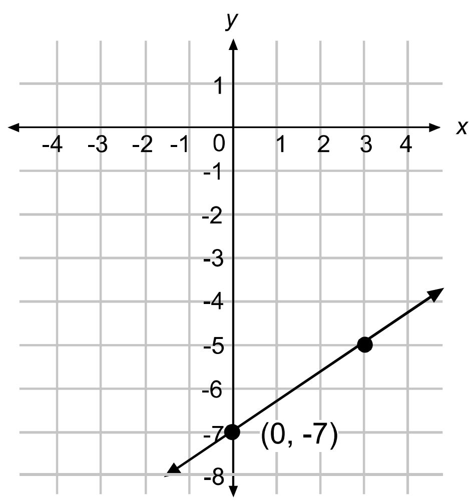
For the equation of the line

$$y = \frac{2}{3}x - 7,$$

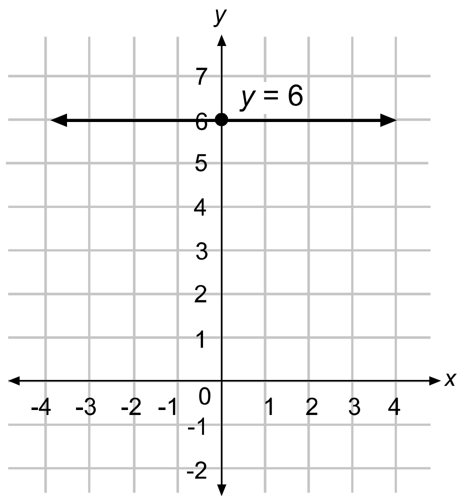
the y-intercept is -7.



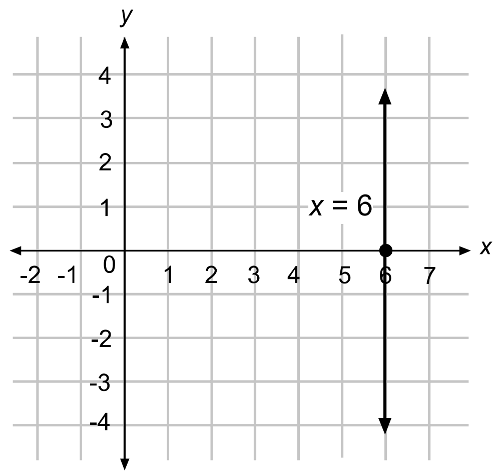
The slope is $\frac{2}{3}$, so the graph looks like the following.



Remember: If the equation looks like $y = 6$, the line is *horizontal* and has *zero* slope. If the equation looks like $x = 6$, the line is *vertical* and has *no* slope. Look at the graphs below.



y = 6
line is horizontal
zero slope



x = 6
line is vertical
no slope
(sometimes referred to as undefined)

Transforming Equations into Slope-Intercept Form

Sometimes it is necessary to transform an equation into the *slope-intercept form* so that we can readily identify the slope or the y-intercept or both.

Follow these examples. Remember, we want it to be in the $y = mx + b$ format.

Example 1

$$\begin{aligned} 6x - 3y &= 12 \\ -3y &= -6x + 12 && \longleftarrow \text{ subtract } 6x \text{ from both sides} \\ y &= 2x - 4 && \longleftarrow \text{ divide both sides by -3} \end{aligned}$$

Now we can easily see that the slope is 2 and the y-intercept is -4.

Example 2

$$\begin{aligned} x + \tfrac{2}{3}y &= 8 \\ \tfrac{2}{3}y &= -x + 8 && \longleftarrow \text{ subtract } x \text{ from each side} \\ (\tfrac{3}{2})\tfrac{2}{3}y &= -(\tfrac{3}{2})x + (\tfrac{3}{2})8 && \longleftarrow \text{ multiply both sides by } \tfrac{3}{2} \\ y &= -\tfrac{3}{2}x + 12 && \longleftarrow \text{ simplify} \end{aligned}$$

slope = $-\frac{3}{2}$ y-intercept = 12